

A Comparison of Three Elliptical Galaxy Photo-chemical Evolution Codes

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ABSTRACT

Working within the classic supernovae-driven wind framework for elliptical galaxy evolution, we perform a systematic investigation into the discrepancies between the predictions of three contemporary codes – Arimoto & Yoshii (1987, A&A, 173, 23), Bressan et al. (1994, ApJS, 94, 63), and Gibson (1996a, MNRAS, 278, 829; 1996b, MNRAS, submitted). By being primarily concerned with reproducing the present-day colour-metallicity-luminosity (CML) relations amongst ellipticals, the approaches taken in the theoretical modelling have managed to obscure many of the hidden differences between the codes. Targeting the timescale for the onset of the initial galactic wind t_{GW} as a primary “difference” indicator, we demonstrate exactly how and why each code is able to claim successful reproduction of the CML relations, despite possessing apparently incompatible input ingredients.

Subject headings: galaxies: abundances — galaxies: elliptical and lenticular, cD — galaxies: evolution — methods: miscellaneous

1. Introduction

In one of his early seminal papers, Larson (1974) postulated that the mass-metallicity relation observed in the present-day elliptical galaxy population (e.g. Faber 1977) was a natural consequence of supernovae (SNe)-driven galactic winds. With each SN contributing $\sim 10^{50}$ erg of thermal energy to a galaxy’s interstellar medium (ISM), Larson demonstrated that standard star formation rate scenarios would inevitably result in the expulsion of any remaining gas once its accumulated thermal energy exceeded that of its gravitational binding energy. The bulk of subsequent star formation would then be suppressed (at this time t_{GW}), thereby “freezing” in the chemical imprints which would be observable today in the stellar populations. Because of a larger binding energy per unit mass, t_{GW} would occur later in more massive systems, thereby allowing the enrichment process to progress further than in more massive systems, in rough accordance with the observed mass-metallicity relation.

The elegant simplicity of Larson’s (1974) model was immediately recognized and provided the basis for a generation of elliptical galaxy SNe-driven wind models. The primary motivation behind the majority of subsequent studies has been to better understand the chemical (e.g. Matteucci & Tornambè 1987; Mihara & Takahara 1994; Matteucci 1994) and dynamical (e.g. Saito 1979b; Dekel & Silk 1986; Ciotti et al. 1991) evolution of these systems. Related to this has been the particular attention paid to the role played by cluster ellipticals in polluting the intracluster medium of galaxy clusters with metals via large scale super-winds (e.g. Larson & Dinerstein 1975; Matteucci & Vettolani 1988; Nath & Chiba 1995; Gibson & Matteucci 1996).

Transcending from the mass-metallicity plane, to a more convenient observational colour-metallicity-luminosity (CML) space, requires the parallel computation of a system’s photometric evolution. This is an important step to take, as the photometric properties

of galaxies should be a primary constraint in any galactic evolution code. Until recently, though, this has been a difficult prospect, owing to the dearth of available self-consistent, metallicity-sensitive stellar evolution tracks and photometric calibrations. This has been alleviated somewhat with the release of the Kurucz (1993) grid of model atmospheres. Their subsequent use, in particular by Worthey (1994) and Bertelli et al. (1994), in constructing extensive grids of isochrones has made the coupled photo-chemical evolution of ellipticals, within the SNe-driven wind framework, a realistic aim.

Indeed, during the past few years, several groups have begun investigations in just this direction – Bressan et al. (1994) (BCF94, hereafter) and Gibson (1996b)¹ (G96, hereafter) are the first of the new generation of SNe-driven wind codes to take advantage of the coupled photometric and chemical evolution possibilities. A precursor, in the same vein as these two more recent codes, belongs to Arimoto & Yoshii (1987) (AY87, hereafter). Their efforts in piecing together, from innumerable sources, a grid of stellar evolution tracks and photo-chemical calibrations, was nothing less than heroic.

In the course of testing our own code, it has become apparent that for all the similarities in their goals (i.e. replicating the present-day CML relations, primarily) and approaches (i.e. classic SNe-driven wind model of Larson 1974), there are more than a few subtle, and “hidden”, differences in the above three packages which have not been fully appreciated. In this paper, we attempt to bring to light, in as straightforward a manner as possible, some of these hidden differences. Considering the proliferation of such models, we feel it imperative (and long overdue) that such a presentation be undertaken.

In Section 2.1, we briefly describe a simple template model with which to work,

¹ Preliminary results can be seen in Gibson (1994a,b;1995;1996a) and Gibson & Matteucci (1996).

drawing attention to each of the intrinsic input ingredients (e.g. supernova remnant (SNR) ISM thermal energy deposition efficiency, nucleosynthetic yields, initial mass function (IMF)). The template chosen is from G96, and successfully reproduces the present-day photo-chemical properties. We then systematically explore the influence of varying each of the primary input ingredients, in accordance with those chosen by AY87 and BCF94. This is done in Sections 2.2 and 2.3, respectively. This is an important exercise which has never been carried out before, and serves to illustrate exactly how the different groups were able to piece together seemingly incompatible ingredients and yet, apparently, recover the proper present-day observable photo-chemical properties. A summary is provided in Section 3.

2. Comparison With Recent Work

Let us re-iterate once again that the general framework adopted by the three groups – AY87, BCF94, and G96 – is quite similar. Specifically, elliptical proto-galaxies are taken to be initially homogeneous gas spheres, and the chemical evolution proceeds according to the simple closed-box model. Each avoids the instantaneous recycling approximation, but subscribes to the instantaneous mixing approximation. The global thermal energy budget of the ISM is assumed to follow the basic prescription outlined in Saito (1979b) and global ejection of said ISM is taken to occur at t_{GW} (i.e. the point at which the thermal energy exceeds the gravitational binding energy).

Each of the groups claim reproducibility of the fundamental CML relations for ellipticals. This is a most interesting result given the very different assumptions regarding star formation rate efficiencies, dark matter distributions, SNe progenitor assumptions, nucleosynthesis, IMF, SNe thermal energy deposition to the ISM efficiency, etc. The combination of all these differences manifests itself in very different predictions for t_{GW} .

AY87 and BCF94 represent the extremes in this apparent dichotomy; for massive ellipticals, the former prefer the late-time galactic wind ($t_{\text{GW}} \approx 1$ Gyr), whereas the latter prefer a much earlier time-scale ($t_{\text{GW}} \approx 0.1$ Gyr). On the surface, these results would appear incompatible, particularly when one considers that star formation is presumed to occur only for $t \lesssim t_{\text{GW}}$. That both groups can claim their final models match the observations, when one’s star formation is an order of magnitude longer in duration, is puzzling, and it is important to be aware of exactly how this comes about in order to appreciate the claims of all the groups in question.

To anticipate what follows, let us look at one massive elliptical scenario: for an initial gas mass $M_g(0) = 10^{12} M_\odot$ and Salpeter (1955) IMF, G96 predict $t_{\text{GW}} = 0.44$ Gyr, whereas for the same IMF slope, BCF94 find $t_{\text{GW}} = 0.09$ Gyr. Both the final colours (e.g. $V-K \approx 3.35$) and metallicity ($[< Z >]_V \approx +0.45$) match the observations. On the other hand, for a flatter IMF (e.g. power-law slope, by mass, of $x = 0.95$), G96 found $t_{\text{GW}} = 0.05$ Gyr, whereas for the same slope, AY87 found $t_{\text{GW}} = 0.71$ Gyr. Again, both the colours and metallicities were consistent with those just mentioned. One is left with the question – *what are the driving factors which lead to very different results for t_{GW} , and yet still allow each group to claim reproducibility of the present-day observational constraints?*

At some level, this is difficult to answer as there is a tendency to not report each and every assumption regarding the various input ingredients. As such, we do not wish to belabour the issue by minutely examining every parameter of these codes, but we do feel it important to highlight some of the primary differences which lead to the, at times, very different results.

2.1. Gibson (1996b)

Our working template, as described in detail in Gibson (1995,1996b), was generated using the time- and metallicity-independent Salpeter (1955) IMF, with lower and upper mass limits of $m_\ell = 0.2 \text{ M}_\odot$ and $m_U = 65.0 \text{ M}_\odot$, respectively. The metallicity-dependent yields of Woosley & Weaver (1995) were used for Type II SNe. The global thermal evolution of the ISM was governed by knowledge of the Type Ia and II SNe rates as a function of time, coupled with the thermal energy made available to the ISM by each SN, as a function of time. The model for the latter assumes that individual SNRs halt their expansion once coming into pressure equilibrium with the ambient ISM, but continue to cool radiatively *ad infinitum*. This form is denoted Model B₂ in Gibson’s (1994b) notation and is taken directly from Cioffi et al. (1988). Diffuse dark matter halos with mass and radial extent ratios relative to the luminous component of ten were used, following the formalism of Bertin et al. (1992). 3% of the mass in the IMF in the mass range $3 \rightarrow 16 \text{ M}_\odot$ was assumed to be locked into Type Ia-progenitor binary systems (Greggio & Renzini 1983), an *a posteriori* choice which ensured a present-day Type Ia SN rate in agreement with Turatto et al. (1994)². Star formation was assumed to proceed in lockstep with the available gas mass, the proportionality constant linking the two, ν , simply being the inverse of the time scale for star formation. The chemical evolution is similar in spirit to that of Matteucci & Greggio (1986), although we have adopted a more intuitively obvious “mass in/mass out” formalism (Timmes et al. 1995), as opposed to the Talbot & Arnett (1971) “matrix” formalism. The photometric evolution was coupled to the chemical evolution, as outlined in Gibson (1996a). In general, the metallicity-dependent isochrones of Worthey (1994) were adopted, although when those of Bertelli et al. (1994) were used, the distinction is made.

² $H_0 \equiv 50 \text{ km/s/Mpc}$ assumed throughout.

Table 1 shows some of the properties of our template of models – in the first block (Salpeter 1955 IMF), five different initial gas masses (column 1) are shown. We draw attention to the star formation efficiency ν in column 2; for the chosen set of input ingredients, ν was treated as a free parameter (as it was in AY87 and BCF94), the value shown ensured that the present-day CML relationships were recovered. The masses of gas, oxygen, and iron ejected at time t_{GW} (column 3) are listed in columns 4, 5, and 6, respectively. The present-day absolute V-band magnitude and V-K colour are given in columns 7 and 8. Column 9 gives the predicted V-band luminosity-weighted metallicity $[< Z >]_{\text{V}}$. The stellar populations of the giant ellipticals in our template have $[\text{Mg}/\text{Fe}]_* \approx +0.15$, which is only marginally lower than the observed $\sim +0.2 \rightarrow +0.3$ (Worthey et al. 1992).

The final entry in Table 1 illustrates how choosing the flatter IMF of slope $x = 0.95$ necessitates increasing ν by a factor of ~ 5 in order to ensure that the colours do not become too red, nor the metallicity too high. We will return to this shortly in Section 2.2. As an aside, the flatter IMF leads to an increased value for $[\text{Mg}/\text{Fe}]_*$ of $\sim +0.35$.

In Figure 1, we show the evolution of the elemental abundance of the primary metals³ for the $M_{\text{g}}(0) = 10^{12} \text{ M}_{\odot}$ template model of Table 1. The evolution shown compares favourably⁴ with that shown in Figure 1 of Matteucci & Padovani (1993). Recall that star formation in this model ceases at $t_{\text{GW}} = 0.44 \text{ Gyr}$. One can see that it only takes $\sim 0.1 \text{ Gyr}$

³For element X, $[\text{X}] \equiv \log X - \log X_{\odot}$.

⁴The abundance “spike” immediately following t_{GW} in Figure 1 is not seen in Matteucci & Padovani’s (1993) Figure 1. This is simply an artifact of the two codes’ temporal resolutions; in the vicinity of t_{GW} , Matteucci & Padovani use $\Delta t \approx 50 \text{ Myr}$, whereas our curves were generated with a uniform $\Delta t = 0.5 \text{ Myr}$. The two codes are entirely consistent when identical Δt values are assumed.

Table 1. G96 Template Models^a

$M_g(0)$	ν	t_{GW}	m_g^{ej}	m_{O}^{ej}	$m_{\text{Fe}}^{\text{ej}}$	M_V	V-K	$[< Z >]_V$
IMF Slope $x = 1.35$								
1.0e6	188.9	0.006	3.5e5	2.2e2	1.0e1	-8.21	2.08	-2.28
5.0e7	209.7	0.007	1.2e7	3.2e4	1.5e3	-12.61	2.12	-1.56
1.0e9	123.1	0.016	1.7e8	3.0e6	2.0e5	-15.89	2.44	-0.51
5.0e10	46.0	0.077	3.4e9	1.1e8	9.3e6	-20.15	3.04	+0.13
1.0e12	17.3	0.440	1.7e10	4.9e8	7.5e7	-23.45	3.33	+0.44
IMF Slope $x = 0.95$								
1.0e12	88.3	0.049	9.6e10	6.6e9	6.2e8	-23.03	3.38	+0.44

^a Input ingredients: Type Ia and II SNe yields from Thielemann et al. (1993) and Woosley & Weaver (1995); low mass stellar yields from Renzini & Voli (1981). Photometric properties (columns 7-9) are based upon Worthey’s (1994) isochrones. SNR energetics governed by Model B₂ of Gibson (1994b). The inverse of the star formation timescale ν [Gyr⁻¹] is in column 2. Column 3 shows the galactic wind time t_{GW} [Gyrs], for the relevant initial gas masses (column 1 – M_{\odot}). The masses of gas, oxygen, and iron ejected at t_{GW} are listed in columns 7-9 [M_{\odot}]. See Section 2.1 for details.

for any one of the elements to reach their approximate solar abundance (within a factor of ~ 2). Note that each of the elements undergoes a post- t_{GW} dilution due to the continually increasing gas mass (from dying low mass stars). The one element which is immune to this dilution is iron – the enormous quantity of Fe ejected per SNe Ia event ($\sim 0.74 M_{\odot}$ – Thielemann et al. 1993) is enough to counteract said dilution. Carbon declines slightly slower than the other elements (besides iron, of course) after t_{GW} due to the increased importance of the carbon-producing single low and intermediate mass stars (Renzini & Voli 1981). As the Type II yields used for this model (Woosley & Weaver 1995) did not have any primary nitrogen production, it is not until the $\sim 4 \rightarrow 8 M_{\odot}$ stars with primary N (i.e. the Renzini & Voli 1981 models with hot bottom burning included) start dying that [N] starts increasing. This is reason why the [N] curve takes ~ 50 Myr to approach that of the others.

EDITOR: PLACE FIGURE 1 HERE.

2.2. Arimoto & Yoshii (1987)

Recall from Section 2 that adopting an IMF slope $x = 0.95$, and adjusting ν to ensure the proper CML predictions for the $10^{12} M_{\odot}$ model are recovered, leads to $t_{\text{GW}} = 0.05$ Gyr, considerably smaller than the $t_{\text{GW}} = 0.71$ found by AY87, yet both claim to replicate the present-day photo-chemical properties for said elliptical. Let us now step through, one-by-one, the sources and implications of the discrepancy. As a starting point, we use the G96 prediction, given by the final entry to Table 1:

- *Star Formation Efficiency:* For the $10^{12} M_{\odot}$ model, AY87 required $\nu = 8.6 \text{ Gyr}^{-1}$ to recover the proper present-day photo-chemical properties, whereas we needed $\nu = 88.3 \text{ Gyr}^{-1}$. If we were to reduce ν by a factor of ten, our predicted t_{GW} would increase

from 0.05 Gyr to 1.42 Gyr. The predicted (V-K) and $[< Z >]_V$, with this change alone, though, would both be erroneous (specifically, 4.04 and +0.83, respectively – recall that $V-K \approx 3.35$ and $[< Z >]_V \approx +0.45$ were the best values for this luminosity).

- *Type Ia SNe:* AY87 neglect Type Ia SNe entirely. This immediately eliminated one potential observational constraint from their modeling (i.e. the magnesium overabundances relative to iron – Worthey et al. 1992), as this all-important iron source was not included. On the other hand, this is not particularly important for t_{GW} ; when using their flat IMF to run the previous model, but this time with Type Ia SNe “switched off”, t_{GW} only increases to 1.43 Gyr.
- *Yields:* AY87 use the older Arnett (1978) yields, although the exact implementation is unclear (in particular, for $Z > Z_\odot$). Still, replacing the Woosley & Weaver (1995) yields with those of Arnett (1991) (which are reasonably similar to his older 1978 yields, as far as global Z is concerned), led to a mild increase in t_{GW} to 1.51 Gyr. This comes about because the SNR thermal energy evolution Model B₂ is mildly metallicity-dependent at late-times (i.e. $\varepsilon_{th} \propto Z/Z_\odot^{-0.13}$ – Cioffi et al. 1988), and for the same ν , the Arnett (1991) yields lead to a slightly more enriched gas, in comparison with the Woosley & Weaver (1995) yields (Gibson 1995, 1996b). The more enriched gas leads to increased cooling, which decreases the effective energy contribution per SN, and thus increases t_{GW} .
- *Dark Matter:* AY87 do not include any dark matter component. Eliminating this reduces t_{GW} from the previous 1.51 Gyr, to 1.25 Gyr, due to the slight reduction in the depth of the potential well.
- *ISM Binding Energy:* Ignoring dark matter, AY87 used the gaseous binding energy formalism of Saito (1979b), as opposed to that of Bertin et al. (1992), which was used by G96 and BCF94. This older form, in the absence of dark matter, predicts higher

binding energies for the same residual ISM gas mass. Using Saito’s (1979a) form in lieu of Bertin et al. ’s (1992), leads to an increase in t_{GW} from the previous 1.25 Gyr, to 2.06 Gyr.

- *SNe Thermal Energy Evolution:* AY87 use what we (Gibson 1994b) call Model A_0 , which is simply the classic SNR energetics form due to Cox (1972) and Chevalier (1974). On the other hand, G96 use the aforementioned Model B_2 , based upon Cioffi et al. (1988). Switching to A_0 reduces t_{GW} to 1.82 Gyr. Model A_0 neglects any of the metallicity effects in the cooling of SNRs, so, as noted above, it is to be expected that there is a marginal increase in SNR energy efficiency, leading to a slightly earlier wind (at least for predominantly super-solar metallicity populations, such as the giant elliptical currently under consideration). Still, as noted by Gibson (1994b), Models A_0 and B_2 are quantitatively similar, despite their somewhat different approaches, which is why the influence on t_{GW} is not excessive.
- *Stellar Lifetimes:* AY87 use the Talbot & Arnett (1971) singular power-law for stellar lifetimes, which tends to overestimate, by up to an order of magnitude, the lifetime of most Type II SNe progenitors. Using this form for the lifetimes, instead of the more appropriate Schaller et al. (1992) ones, increases t_{GW} again, from 1.82 Gyr to 2.18 Gyr.
- *Hydrogen Number Density - Further SNe Energetics:* AY87 assumed that the hydrogen number density n_0 was constant throughout time, and given by $n_0(t = 0)$. This overestimates n_0 for $t > 0$, which in turn, depletes the available energy per SN event which is made available to the ISM for powering a galactic wind, because of the $\sim n_0^{-1/3}$ dependence in the late-time SNR interior thermal energy evolution (Chevalier 1974). Adopting their invalid assumption results in a large increase in t_{GW} , from 2.18 Gyr, to 10.52 Gyr. Such a late wind epoch seems highly unlikely, as, under the

current formalism, the implied star formation rates for ellipticals in the redshift range $z \approx 0.1 \rightarrow 0.4$ would be $\sim 30 \rightarrow 40 \text{ M}_\odot/\text{yr}$, inconsistent with the observed rates (e.g. Sandage 1986). This error was also discussed by Angeletti & Giannone (1990).

- *IMF Limits:* Instead of our template IMF range of $0.2 \rightarrow 65.0 \text{ M}_\odot$, AY87 use $0.05 \rightarrow 60.0 \text{ M}_\odot$. This has the advantage of tying up much more mass in ultra-low mass (i.e. long-lived) objects, thereby reducing the gaseous binding energy at later times, despite the corresponding reduction in the absolute Type II SNe production. Adopting their mass range reduces t_{GW} from 10.52 Gyr, to 7.81 Gyr.
- *Type II SNe:* To partially compensate for the lack of Type Ia SNe in their modelling, AY87 arbitrarily adopt a lower limit for Type II SNe progenitors of 3.0 M_\odot . This is, of course, incorrect on stellar evolution grounds, but obviously serves as a convenient means to reduce t_{GW} to more reasonable values (as all the $3 \rightarrow 8 \text{ M}_\odot$ stars which would normally end their lives as thermally pulsating asymptotic giant branch stars would now be assumed to end their lives in a SN explosion) – specifically, t_{GW} is now predicted to be 0.98 Gyr. At this point, the predicted V-K colour and metallicity $[< Z >]_{\text{V}}$ are 3.89 and +0.78, respectively, which are both at odds with the observed $\text{V-K} \approx 3.35$ and $[< Z >]_{\text{V}} \approx +0.45$.
- *Proto-galactic Radius:* In G96, as in most other galactic wind codes, the radius of the pre-wind proto-galaxy is taken to be approximately that of the present-day spheroids. At some level this is probably incorrect (e.g. Hills 1980; Dekel & Silk 1986; Angeletti & Giannone 1991; Elbaz et al. 1995), but for lack of better observational constraints on proto-galactic sizes at the onset of star formation, this seems the most conservative approach. In the published AY87 paper, this also appears to be the approach taken. In that case, our previously mentioned $t_{\text{GW}} = 0.98 \text{ Gyr}$ should be directly comparable to their value of 0.71 Gyr. The situation becomes slightly

unclear, though, when we refer back to their erratum (Arimoto & Yoshii 1989) in which it is stated that the proto-galactic radius R_L used was actually a factor of two larger than that predicted by the Saito (1979a) mass-radius relations. This, in turn, implies that the proto-galactic binding energy is a factor of two smaller. Assuming that to be the case, if we re-run the last model, we find $t_{\text{GW}} = 0.28$ Gyr. Equally confusing, the preprint version of the AY87 paper states that they actually adopted a proto-galactic radius which was 4.74 times that given by Saito (1979a)⁵ Adopting this more extreme initial condition, leads to $t_{\text{GW}} = 0.03$ Gyr, again due to the reduction in the depth of the potential well. The confusing nature of the proto-galactic radius used by AY87 makes replicating their results, exactly, difficult. Still, based upon the fact that their published $t_{\text{GW}} = 0.71$ Gyr, is closer to what we found when simply using Saito’s (1979a) binding energy prediction ($t_{\text{GW}} = 0.98$ Gyr), as opposed to what was found using arbitrary reductions of factors of $\sim 2 \rightarrow 5$, it would seem likely that this is what AY87 assumed.

- *Photometric Calibrations:* AY87 (as described in more detail in Arimoto & Yoshii 1986) did not have access to any super metal-rich (i.e. $Z > Z_\odot$) photometric calibrations). As such, the best they could do was assume that metal-rich dwarfs and giants obeyed the same colour-luminosity relations as $Z = Z_\odot$ stars. For the dwarf galaxies, whose ISMs do not exceed $Z \sim Z_\odot$ for most of their star forming period, this is not a problem. For the giant elliptical under consideration in this comparison, though, the bulk of the star formation (i.e. $t \gtrsim 0.05$ Gyr – see Table 5 of AY87) occurs while the ISM metallicity is super-solar, approaching ten times solar after a few tenths

⁵This factor of 4.74, as quoted in the AY87 preprint, led Matteucci & Tornambè (1987) to do the same. This was corrected for subsequent papers based upon her code (Matteucci 1995). Angeletti & Giannone (1990) have likewise commented on this factor of 4.74.

of a Gigayear. As noted above, if we use the full grid of Worthey (1994) isochrones, including properly the super-solar metallicity ones, to predict the integrated V-K of the stellar populations at $t = 12$ Gyr, we find $V-K \approx 3.89$, which is ~ 0.55 mag redder than the mean observed value for this luminosity. This would imply that t_{GW} is too late for the given initial mass/star formation scenario. On the other hand, if we impose the condition that all $Z > Z_{\odot}$ stars obey the $Z = Z_{\odot}$ colour-temperature scale, we would actually predict $V-K \approx 3.28$, very much in line with both the observed mean, and that predicted by AY87.

- *Luminosity-weighted Metallicity:* Instead of calculating the mean luminosity-weighted metallicity $\langle [Z] \rangle_V$, AY87 use the mean of the logarithmic, luminosity-weighted metallicity $\langle [Z] \rangle_V$. This is somewhat incorrect, as spectral indices scale much closer to Z (i.e. number of absorbers) than to $\log Z$ (e.g. Worthey 1994; González & Gorgas 1995). A simple numerical example illustrating how this could lead to an underestimation of the true stellar metallicity is shown in Gibson (1996a). AY87 do not include any post-red giant branch contribution (i.e. no horizontal branch, asymptotic giant branch, white dwarfs, etc.), which will play a part at some level, especially the lack of AGB stars, a point which they themselves acknowledge in their paper.

In summary, we feel confident that we now have a good understanding of the source of the majority of differences in the predictions of AY87 and G96; of particular importance would appear to be the galactic wind time/photometric calibration “conspiracy”.

For $M_g(0) = 10^{12} M_{\odot}$, G96’s preferred Salpeter (1955) IMF model predicted $t_{\text{GW}} = 0.44$ Gyr, which is equivalent to $\sim 3 \times 10^{48}$ erg/SN being made available to the ISM thermal energy reservoir. In comparison, AY87 favoured a much flatter IMF ($x = 0.95$) and $t_{\text{GW}} = 0.71$ Gyr, which still corresponds to $\sim 2 \times 10^{48}$ erg/SN. In fact, AY87’s models

required both a significantly flatter-than-Salpeter (1955) IMF, and the late galactic wind time, in order to ensure that enough of the stars formed during the “super-solar” metallicity phase sampled the $Z=Z_{\odot}$ stellar evolution tracks and their respective photometric calibrations. Our models show that AY87’s *necessary* high mass star biased IMF is no longer a necessity, provided one takes account of the $Z>Z_{\odot}$ photometric calibrations properly.

On the other hand, if we do impose the $x = 0.95$ IMF⁶, without altering the star formation efficiency, we find the final colours/metallicity become too red/rich ($V-K=3.97$ and $[< Z >]_V = +0.77$, respectively), as t_{GW} does not change dramatically (Gibson 1995,1996b). For this flatter IMF, by increasing ν by a factor of ~ 10 , one can recover the appropriate photo-chemical properties, with $t_{\text{GW}} = 0.05$ Gyr. In this latter scenario, the effective energy contribution per SN event is found to increase to $\sim 10^{49}$ erg.

In conclusion, AY87 were forced to accept a flatter-than-Salpeter (1955) IMF because of their lack of metal-rich photometric calibrations. This would appear to no longer be a necessity. Also, at some level AY87 were somewhat fortuitous in their recovering the present-day photo-chemical properties within this simple framework, given that the SNR energetics were based upon the $t = 0$ ISM density. Because this would, in general, lead to inordinately late wind epochs, the lowering of the Type II SN-progenitor lower mass limit to $3 M_{\odot}$ was needed in order for the wind epoch to occur at a more feasible value. This is very much an example of two wrongs making a right!

⁶Indeed, such an IMF is advantageous when attempting to account for the intracuster medium abundances (Gibson & Matteucci 1996).

2.3. Bressan, Chiosi & Fagotto (1994)

A more recent addition to coupled “elliptical galaxy photo-chemical evolution” studies, with galactic winds, comes from BCF94. Their models are similar to AY87, with the primary difference being a substantial improvement in the input stellar physics. Specifically, BCF94 are able to draw upon their impressive “in-house” expertise in stellar evolution theory, generating a self-consistent grid of stellar tracks from the zero age main sequence to the white dwarf/SN stage, for a wide range of masses and metallicities (Bertelli et al. 1994). Super-solar photometric calibrations are provided by the newly-available Kurucz (1993) model atmospheres (with the odd empirical extension).

BCF94 do not list their yield sources, so one must bear this mind when comparing any chemical evolution models. A more serious problem lies in attempting to replicate their photometric predictions. It was presumed that the Bertelli et al. (1994) metallicity-dependent isochrones were employed “as is”, yet comparing the integrated colours of the simple stellar populations (SSPs) in the BCF94 paper (their Table 3) with those in the original isochrone paper of Bertelli et al. (1994), shows that some unspecified modifications were made for the BCF94 analysis. This has now been corroborated by Charlot et al. (1996), and further quantified by Gibson (1996a).

Let us first just list some of the relevant input ingredients to their models. Minor differences in G96 and BCF94 can be seen here, and are ignored for the discussion which follows; we shall only concern ourselves with the most important ones in the itemized list which follows below: SNR thermal energy in the ISM follows the older Model A₀ (Cox 1972; Chevalier 1974; Gibson 1994b); a Salpeter (1955) IMF with $m_\ell = 0.1 M_\odot$ and $m_U = 120.0$ is adopted; the ratio of dark-to-luminous mass and radial extents is taken to be five, and the Bertin et al. (1992) formalism followed; the star formation efficiency ν is assumed to be 20 Gyr^{-1} , independent of galactic mass; we shall assume throughout that the Arnett (1991)

yields were used for the Type II SNe ejecta.⁷

Recall from Section 2 that for $M_g(0) = 10^{12} \text{ M}_\odot$, BCF94 found $t_{\text{GW}} = 0.09 \text{ Gyr}$ was necessary to recover the proper present-day photo-chemical properties. Contrast this with the 0.44 and 0.71 Gyr found, respectively by G96 and AY87. As we will show now, the comparison which we undertook with BCF94, resulted in the identification of one or two potential problem areas. Note that in places we ignore the small differences in t_{GW} invoked by input ingredient variations. Changes at the level shown in the corresponding items of Section 2.2 are to be expected.

- *SNe Progenitors:* Unlike AY87, BCF94 do include Type Ia SNe, but only in the calculation of the ISM energetics. Their role in enriching the ISM (and in particular the iron abundance) is not considered. Obviously then, they are not concerned with predicting $[\text{Mg}/\text{Fe}]_*$ in the resultant stellar populations, nor with the predicted $[\alpha/\text{Fe}]$ ratio for the intracluster medium.
- *Stellar Metallicity Determination:* BCF94 do not present luminosity-weighted metallicities, but opt for a mass-weighted determination. This is usually the recourse for codes which do not have a parallel photometric evolution code (e.g. Matteucci & Tornambè 1987; Angeletti & Giannone 1990; Mihara & Takahara 1994; Elbaz et al. 1995). As noted in AY87 and Gibson (1996a), this can, in some instances, lead one to overestimate the true metallicity of the system, although this does not impact on the results which follow.
- *Pre-SN Stellar Wind Energetics:* BCF94 have adopted what we feel is an unrealistic energy formalism for stellar wind energy deposition to the ISM. In fact, in the BCF94

⁷This assumption is based solely on the fact that some of the earlier papers from the Padova Group used yields based upon the related Arnett (1978) compilation.

models, SNe are a completely inconsequential component to ISM energetics, and it is the enormous stellar wind energy which leads to their consistently small values for t_{GW} . This conclusion is at odds with all the previously mentioned galactic wind studies. We realize that BCF94 *needed* to impose an early (i.e. $t_{\text{GW}} \lesssim 0.1$ Gyr) galactic wind in order to recover the present-day elliptical galaxy CML relations (see next item below), but from a physical standpoint their arguments do not seem sound, as discussed already in Gibson (1994a)⁸. Regardless of this physics, Bressan et al. (1996) and Tantaló et al. (1996) still maintain that stellar wind energy of this magnitude is a necessity.

We do not want to repeat the argument of Gibson (1994a) here, but there are two comments which should be made – *First*, early galactic wind times of the sort promoted by BCF94 are entirely feasible with the standard *SNe*-driven scenario. All of the individual SNR models talked about thus far (e.g. Models A₀ and B₂ of Gibson 1994b) assume shells expand in isolation *ad infinitum*. As we saw in previous sections, this resulted in the effective energy contribution to the ISM per SN being of the order 10^{48} erg. There is certainly no reason to expect that such a behaviour is necessarily the correct one. As Larson (1974) himself noted, in reality, shells will come into contact and merge/overlap with neighbouring shells, forming large superbubbles (e.g. Tomisaka 1992). Subsequent SNe will continue to explode inside the low density medium behind the expanding superbubble, and as such suffer far less from radiative losses. This can easily raise the effective energy contribution per SN from $\sim 10^{48}$ erg to $\gtrsim 10^{49}$ erg. Larson (1974) claims a value of $\sim 10^{50}$ is actually more appropriate. This, perhaps, more realistic SNR evolution model is denoted Model B₃' in Gibson's

⁸Although we readily admit that stellar wind energetics can play a non-negligible role in powering galactic winds in dwarf galaxies of mass $M \lesssim 10^9 M_{\odot}$ (Gibson 1994a).

(1994b) notation. A template of models (parallel to that shown in Table 1) using such an SNR evolutionary scenario has already been presented in Gibson & Matteucci (1996), and will not be discussed further here. The *second*, and primary reason for not arguing the pros and cons of the different SNR evolution models here, is that there appears to be a potential error in the BCF94 chemical evolution code, which, when accounted for, would seem to remove the necessity for an early galactic wind in their study. Let us discuss this further in the following item, as it is of primary importance to interpreting the BCF94 results.

- *Chemical Evolution:* Figure 9 of BCF94 shows the evolution of the ISM metallicity Z_g , for their favoured suite of models. With the identical input parameters, we ran both our code (G96) and that belonging to Matteucci (1992) (hereafter, M92), in order to compare against the BCF94 code, the results of which are shown in Figure 2. The top panel shows the evolution of the system gas mass fraction ($f_g \equiv M_g(t)/M_g(0)$). Consistency is seen between each of the three codes. It is the bottom panel which indicates that something *may* be amiss with BCF94’s chemical evolution code⁹. If,

⁹We feel reasonably confident that the “problem” does not lie in the G96 and M92 codes. Both use very different approaches to solving the chemical evolution equations, and were developed entirely independent of one another. Further, thanks to the kindness of Leticia Carigi (CIDA) and Frank Timmes (Chicago), the author was able to compare the chemical evolution of their codes (Carigi 1994 and Timmes et al. 1995, respectively) with those of G96 for a variety of well-defined sample cases. As for the M92 and G96 curves of Figure 2, the consistency was excellent. It seems unlikely that all four of these chemical evolution codes have made the same error, and one is left with the impression that the problem lies in the BCF94 code. It is difficult to quantify the problem beyond this level without more knowledge concerning the nature of the BCF94 yield implementation.

though, we were to artificially increase the yields by $\sim 65\%$, then both the G96 and M92 curves would overlay precisely BCF94's. How overestimating the yields by $\sim 65\%$ would impact upon BCF94's conclusions proves most interesting.

EDITOR: PLACE FIGURE 2 HERE.

Figure 3 shows the predicted metallicity-luminosity (upper panel) and colour-luminosity (lower panel) behaviour from two different observational sources. The data in the upper panel is from Terlevich et al. (1981), and that in the lower panel is derived from Bower et al. (1992). Overlain on each data-set are several model predictions.

EDITOR: PLACE FIGURE 3 HERE.

Curve 2 of Figure 3 shows graphically the predictions of G96's template (Table 1). Recall that the choice of the star formation efficiency parameter ν shown in the table ensured the models agreed with the observations. Recall, also, that the galactic wind time t_{GW} was set solely by the SNR energetics (i.e. no pre-SN mass-loss thermalized kinetic energy), and for giant ellipticals, $t_{\text{GW}} \approx 0.4$ Gyr.

Now, if we assume, as Figure 2 suggests, that BCF94 have overestimated their yields by $\sim 65\%$, we can predict what sort of CML relation BCF94 *would* have found, *if* they had only used the conventional Model A₀ SNR energetics, with no additional pre-SN stellar wind energy, along with the incorrect yields. For their $\nu = 20 \text{ Gyr}^{-1}$ (and Salpeter 1955 IMF with $m_{\ell} = 0.1 \text{ M}_{\odot}$ and $m_{\text{U}} = 120.0 \text{ M}_{\odot}$), said prediction is given by curve 1 in Figure 3, with $t_{\text{GW}} \approx 0.4$ Gyr, for the massive ellipticals, as in G96. First, the relations are too flat (particularly in the colour-luminosity plane), but that simply reflects their adoption of a mass-independent value for ν . Second,

and much more importantly, the predicted metallicities and V-K colours are $\gtrsim 0.4$ dex ($\gtrsim 0.4$ mag) too high (red). This demonstrates exactly why BCF94 needed to invoke non-standard stellar wind energetics (recall Gibson 1994a), in order to get the galactic wind to occur early enough so that enrichment could not drive the relations too rich/red.

Curve 4 resembles curve 1 in that the Arnett (1991) yields have been scaled upwards by $\sim 65\%$, *but* we have imposed the galactic wind times, as given by BCF94, to each of the models. These should be the direct analog of BCF94’s models. These latter models are denoted as curve 3 in Figure 3. The metallicity-luminosity agreement between the two is excellent (especially when we consider that BCF94 used a mass-weighted metallicity determination in lieu of a luminosity-weighted one). The (V-K)– M_V relation given by curve 4 is offset by ~ 0.08 mag from curve 3, because of the redder giant branch in Worthey’s (1994) $Z=Z_\odot$ isochrones. This well-known effect has already been discussed by Gibson (1996a) and Charlot et al. (1996). One can now see, especially from the lower panel of Figure 3, why BCF94 preferred their early wind formalism, as it gives the appearance of solving the problem presented by curve 1. We presume that BCF94 were only concerned with recovering the colour-luminosity relation in the bottom panel, as the metallicity-luminosity relation of the upper panel for their curve 3 (or curve 4, for that matter), lies below the Terlevich et al. (1981) calibration. On the other hand, the Terlevich et al. (1981) calibration may overestimate the true metallicity by $\sim 0.1 \rightarrow 0.2$ dex (Barbuy 1994), so we should not overinterpret this particular constraint.

On the other hand, what might we expect if BCF94’s early wind epochs (i.e. dominant pre-SN stellar wind energy) were appropriate, *and* the yields were brought back down in line with Arnett’s (1991) tables? The answer is provided by curve 5 in Figure 3. For the given IMF and star formation efficiency, *if* the yields were accounted for

properly, BCF94 would have found metallicities (colours) which were too low (blue) by ~ 0.5 dex (~ 0.2 mag). We confirmed this with Matteucci’s (1995) mass-weighted metallicity determination code, as applied to models derived from her 1992 chemical evolution code, with enforced wind times to match those of BCF94. Curve 6 of Figure 3 illustrates this result (which parallels that of curve 5, as it should).

In summary, BCF94, Bressan et al. (1996), and Tantalo et al. (1996) have each stressed that the primary motivation for invoking early galactic winds (i.e. $t_{\text{GW}} \lesssim 0.1$ Gyr) via thermalized kinetic energy from pre-SN mass-loss, was not necessarily a “physical” one, but more out of necessity, in order to recover the present-day CML relations. As we have demonstrated above, what drove them to this conclusion appears to have been an overestimation of the metal yields (by $\sim 65\%$) in their chemical evolution code. Correcting this apparent problem removes their entire motivation for looking for an energy source capable of powering early galactic winds in the first place!

This is further corroborated by the downward “extensions” to curve 1 of Figure 3. Here the $10^{12} M_{\odot}$ models are shown by the asterisks, along curve 1 in both panels, where BCF94 would have predicted it to be, assuming no stellar wind energy and Arnett (1991) yields scaled upwards by $\sim 65\%$. Again, the t_{GW} is ~ 0.4 Gyr, in this case. *If* BCF94 had simply retained the SN energetics, but used the Arnett (1991) yields, as is, the predictions would shift to the lower part of the respective extensions. As one can see, the values are in-line with the observations, and any further “tweaking” could be accomplished by modifying ν upwards, slightly. Once again, the point to be made is that “late” galactic winds (i.e. $t_{\text{GW}} \approx 0.4$ Gyr) are more than adequate to recover the CML relations, *provided* the chemical evolution properly reflects the SNe yield predictions.

3. Summary

A comparison of the galactic wind codes of AY87 and G96 demonstrates quite clearly that AY87’s conclusion that the IMF in elliptical galaxies *must* be flatter-than-Salpeter (1955), with a slope, by mass, of $x = 0.95$, is purely an artifact of missing metal-rich ($Z > Z_{\odot}$) photometric calibrations. Two erroneous assumptions in their model (that the ISM density is not a function of time, and that all stars of mass $m \geq 3 M_{\odot}$ end their lives as Type II SNe) fortuitously combine to yield wind times and photo-chemical properties which resemble the present-day observations.

In referring to the early wind predictions of BCF94, it would appear that the criticism of Gibson (1994a) was incomplete. At that point we had argued against pre-SN mass-loss via stellar winds playing such a prominent role in setting t_{GW} , purely from a physical standpoint, *but* that one could still get early winds via proper handling of conventional SN energetics (see also G96 and Gibson & Matteucci 1996). We still stand by these arguments, but, more importantly (and a point which we did not appreciate in the earlier Gibson 1994a), as we have demonstrated in Section 2.3, it would appear that BCF94’s argument for an early wind, in the first place, was predicated upon a substantially different, and perhaps incorrect, treatment of nucleosynthetic yields in their chemical evolution code. Scaling BCF94’s yields down to the tabulated Arnett (1991) values removes the *necessity* for the early wind. It *may* still occur early (i.e. $t_{\text{GW}} \lesssim 0.1$ Gyr) in the evolution (e.g. Gibson & Matteucci 1996), but it is no longer a *necessity*.

More sophisticated models, than those discussed here, are needed before the “early” versus “late” wind-dichotomy can be resolved. Specifically, do SNe contribute $\lesssim 10^{48}$ ergs (late) or $\gtrsim 10^{49}$ ergs (early)? Even knowledge at this level may not be enough to resolve the problem, as it also depends intimately upon the assumed IMF and star formation formalism. For the simple models discussed here, with star formation proportional to the available gas

mass, all we can really say is that a standard Salpeter (IMF) with late (i.e. $t_{\text{GW}} \gtrsim 0.4$ Gyr for giant ellipticals) winds successfully recovers the present-day CML relations, *but* then so do early (i.e. $t_{\text{GW}} \lesssim 0.05$ Gyr) wind models with a flatter-than-Salpeter (1955) IMF of slope $x \approx 1$. One can imagine that other combinations are potentially available. At some level, the arguments of Mihara & Takahara (1994) for simply treating t_{GW} as a free parameter seem appealing, although we still feel it important to at least be aware of the different claims from the groups discussed in this paper, and understand how and why the differences come about.

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Fig. 1.— Time-dependence of the elemental abundances (relative to solar) for the $M_g(0) = 10^{12} M_\odot$, SNe energy Model B₂, template of G96. The galactic wind epoch t_{GW} (and hence point of cessation of star formation) is noted.

Fig. 2.— Evolution of the gas mass fraction f_g (upper panel) and global metallicity Z_g for the $\nu = 20 \text{ Gyr}^{-1}$ models of G96, M92’s chemical evolution code, and BCF94. The Arnett (1991) Type II SNe yields were adopted for the G96 curves. The contribution from SNe Ia is neglected.

Fig. 3.— *Upper Panel:* Mean stellar metallicity versus absolute V magnitude. Observational data from Terlevich et al. (1981). Curve 1: Projected BCF94 behaviour assuming no stellar wind energy contribution, and $1.64\times$ the standard Arnett (1991) yields. Downward extension illustrates the anticipated shift to lower metallicities had BCF94 simply used the tabulated Arnett (1991) yields. Curve 2: G96 behaviour given by Table 1. Curve 3: BCF94’s quoted behaviour, from their Table 5. Curve 4: As curve 1, except with stellar wind energy of the form used in their paper included. Curve 5: As curve 4, except tabulated Arnett (1991) yields adopted. Curve 6: As curve 5, except using mass-weighted metallicity code of Matteucci (1995), as applied to models derived from Matteucci (1992). *Lower Panel:* Optical-Infrared colour versus absolute V magnitude. Observational data from Bower et al. (1992).